

Scheduling step-deteriorating jobs to minimize the total weighted tardiness on a single machine

Peng Guo · Wenming Cheng · Yi Wang

Received: date / Accepted: date

Abstract This paper addresses the scheduling problem of minimizing the total weighted tardiness on a single machine with step-deteriorating jobs. With the assumption of deterioration, the job processing times are modeled by step functions of job starting times and pre-specified job deteriorating dates. The introduction of step-deteriorating jobs makes a single machine total weighted tardiness problem more intractable. The computational complexity of this problem under consideration was not determined. In this study, it is firstly proved to be strongly NP-hard. Then a mixed integer programming model is derived for solving the problem instances optimally. In order to tackle large-sized problems, seven dispatching heuristic procedures are developed for near-optimal solutions. Meanwhile, the solutions delivered by the proposed heuristic are further improved by a pair-wise swap movement. Computational results are presented to reveal the performance of all proposed approaches.

P. Guo
School of Mechanical Engineering, Southwest Jiaotong University, Chengdu, China
E-mail: pengguo318@gmail.com

W. Cheng
School of Mechanical Engineering, Southwest Jiaotong University, Chengdu, China
E-mail: wmcheng@swjtu.edu.cn

Y. Wang
Corresponding author. Department of Mathematics, Auburn University at Montgomery, AL, USA
E-mail: ywang2@aum.edu

1 Introduction

Scheduling with meeting job due dates have received increasing attention from managers and researchers since the Just-In-Time concept was introduced in manufacturing facilities. While meeting due dates is only a qualitative performance, it usually implies that time dependent penalties are assessed on late jobs but that no benefits are derived from completing jobs early [2]. In this case, these quantitative scheduling objectives associated with tardiness are naturally highlighted in different manufacturing environments. The total tardiness and the total weighted tardiness are two of the most common performance measures in tardiness related scheduling problems [31]. In the traditional scheduling theory, these two kinds of single machine scheduling problems have been extensively studied in the literature under the assumption that the processing time of a job is known in advance and constant throughout the entire operation process [18, 34, 9, 7, 4, 35].

However, there are many practical situations that any delay or waiting in starting to process a job may cause to increase its processing time. Examples can be found in financial management, steel production, equipment maintenance, medicine treatment and so on. Such problems are generally known as *time dependent scheduling problems* [11]. Among various time dependent scheduling models, there is one case in which job processing times are formulated by piecewise defined functions. In the literature, these jobs with piecewise defined processing times are mainly presented by *piecewise linear deterioration* and/or *step-deterioration*. In this paper, the single machine total weighted tardiness scheduling problem with step-deteriorating jobs (SMTWTSD) are addressed. For a step-deteriorating job, if it fails to be processed prior to a pre-specified threshold, its processing time will be increased by an extra time; otherwise, it only needs the basic processing time. The corresponding single machine scheduling problem was considered firstly by Sundararaghavan and Kunnathur [32] for minimizing the sum of the weighted completion times.

The single machine scheduling problem of makespan minimization under step-deterioration effect was investigated in Mosheiov [27], and some simple heuristics were introduced and extended to the setting of multi-machine and multi-step deterioration. Jeng and Lin [15] proved that the problem proposed by Mosheiov [27] is NP-hard in the ordinary sense based on a pseudo-polynomial time dynamic programming algorithm, and introduced two dominance rules and a lower bound to develop a branch and bound algorithm for deriving optimal solutions. Cheng and Ding [5] showed the

total completion time problem with identical job deteriorating dates is NP-complete and introduced a pseudo-polynomial algorithm for the makespan problem. Moreover, Jeng and Lin [16] proposed a branch and bound algorithm incorporating a lower bound and two elimination rules for the total completion time problem. Owing to the intractability of the problem, He et al [14] developed a branch and bound algorithm and a weight combination search algorithm to derive the optimal and near-optimal solutions.

The problem was extended by Cheng et al [8] to the case with parallel machines, where a *variable neighborhood search algorithm* was proposed to solve the parallel machine scheduling problem. Furthermore, Layegh et al [22] studied the total weighted completion time scheduling problem on a single machine under job step deterioration assumption, and proposed a *memetic algorithm* with the average percentage error of 2%. Alternatively, batch scheduling with step-deteriorating effect also attracts the attention of some researchers, for example, referring to Barketau et al [3], Leung et al [24] and Mor and Mosheiov [25]. With regard to the piecewise linear deteriorating model, the single machine scheduling problem with makespan minimization was firstly addressed in [20]. Following this line of research, successive research works, such as Kubiak and van de Velde [19], Cheng et al [6], Moslehi and Jafari [28], spurred in the literature.

However, these objective functions with due dates were rarely studied under step-deterioration model in the literature. Guo et al [13] recently found that the total tardiness problem in a single machine is NP-hard, and introduced two heuristic algorithms. As the more general case, the single machine total weighted tardiness problem (SMTWT) has been extensively studied, and several dispatching heuristics were also proposed for obtaining the near-optimal solutions [30, 17]. *To the best of our knowledge, there is no research that discusses the SMTWTSD problem.* Although the SMTWT problem has been proved to be strongly NP-hard [21, 23] and [29, p. 58], the complexity of the SMTWTSD problem under consideration is still open. Therefore, this paper gives *the proof of strong NP-hardness of the SMTWTSD problem. Several efficient dispatching heuristics are presented and analyzed as well.* These dispatching heuristics can deliver a feasible schedule within reasonable computation time for large-sized problem instances. Moreover, they can be used to generate an initial solution with certain quality required by a meta-heuristics.

The remainder of this paper is organized as follows. Section 2 provides a definition of the single machine total weighted tardiness problem with step-deteriorating

jobs and formulates the problem as a mixed integer programming model. The complexity results of the problem considered in this paper and some extended problems are discussed in Section 3. In Section 4, seven improved heuristic procedures are described. Section 5 presents the computational results and analyzes the performances of the proposed heuristics. Finally, Section 6 summarizes the findings of this paper.

2 Problem description and formulation

The problem considered in this paper is to schedule n jobs of the set $\mathbb{N}_n := \{1, 2, \dots, n\}$, on a single machine for minimizing the *total weighted tardiness*, where the jobs have *stepwise* processing times. Specifically, assume that all jobs are ready at time zero and the machine is available at all times. Meanwhile, no preemption is assumed. In addition, the machine can handle only one job at a time, and cannot keep idle until the last job assigned to it is processed and finished. For each job $j \in \mathbb{N}_n$, there is a *basic processing time* a_j , a *due date* d_j and a given *deteriorating threshold*, also called *deteriorating date* h_j . If the *starting time* s_j of job $j \in \mathbb{N}_n$ is less than or equal to the given threshold h_j , then job j only requires a basic processing time a_j . Otherwise, an extra penalty b_j is incurred. Thus, the *actual processing time* p_j of job j can be defined as a step-function: $p_j = a_j$ if $s_j \leq h_j$; $p_j = a_j + b_j$, otherwise. Without loss of generality, the four parameters a_j , b_j , d_j and h_j are assumed to be positive integers.

Let $\pi = (\pi_1, \dots, \pi_n)$ be a sequence that arranges the current processing order of jobs in \mathbb{N}_n , where π_k , $k \in \mathbb{N}_n$, indicates the job in position k . The *tardiness* T_j of job j in a schedule π can be calculated by

$$T_j = \max\{0, s_j + p_j - d_j\}.$$

The objective is to find a schedule such that the *total weighted tardiness* $\sum w_j T_j$ is minimized, where the weights w_j , $j \in \mathbb{N}_n$ are positive constants. Using the standard three-field notation [12], this problem studied here can be denoted by $1|p_j = a_j \text{ or } a_j + b_j, h_j|\sum w_j T_j$.

Based on the above description, we formulate the problem as a 0-1 integer programming model. Firstly, the decision variable y_{ij} , $i, j \in \mathbb{N}_n$ is defined such that y_{ij} is 1 if job i precedes job j (not necessarily immediately) on the single-machine, and 0 otherwise. The formulation of the problem is given below.

Objective function:

$$\text{minimize } Z := \sum_{j \in \mathbb{N}_n} w_j T_j \quad (2.1)$$

Subject to:

$$p_j = \begin{cases} a_j, & s_j \leq h_j \\ a_j + b_j, & \text{otherwise,} \end{cases} \quad \forall j \in \mathbb{N}_n \quad (2.2)$$

$$s_i + p_i \leq s_j + M(1 - y_{ij}), \quad \forall i, j \in \mathbb{N}_n, i < j \quad (2.3)$$

$$s_j + p_j \leq s_i + M y_{ij}, \quad \forall i, j \in \mathbb{N}_n, i < j \quad (2.4)$$

$$s_j + p_j - d_j \leq T_j, \quad \forall j \in \mathbb{N}_n \quad (2.5)$$

$$y_{ij} \in \{0, 1\}, \quad \forall i, j \in \mathbb{N}_n, i \neq j \quad (2.6)$$

$$s_j, T_j \geq 0, \quad \forall j \in \mathbb{N}_n, \quad (2.7)$$

where M is a large positive constant such that $M \rightarrow \infty$ as $n \rightarrow \infty$. For example, M may be chosen as $M := \max_{j \in \mathbb{N}_n} \{d_j\} + \sum_{j \in \mathbb{N}_n} (a_j + b_j)$.

In the above mathematical model, equation (2.1) represents the objective of minimizing the total weighted tardiness. Constraint (2.2) defines the processing time of each job with the consideration of step-deteriorating effect. Constraint (2.3) and (2.4) determine the starting time s_j of job j with respect to the decision variables y_{ij} . Constraint (2.5) calculates the tardiness of job j with the completion time and the due date. Finally, Constraints (2.6) and (2.7) define the boundary values of variables y_{ij} , s_j , T_j , for $i, j \in \mathbb{N}_n$.

3 Complexity results

This section discusses computational complexity of the problem under consideration. It is generally known that once a problem is proved to be strongly NP-hard, it is impossible to find a polynomial time algorithm or a pseudo polynomial time algorithm to produce its optimal solution. Then heuristic algorithms are presented to obtain near optimal solutions for such a problem. Subsequently, the studied single machine scheduling problem is proved to be strongly NP-hard.

Theorem 1 *The problem $1|p_j = a_j \text{ or } a_j + b_j, h_j| \sum w_j T_j$ is strongly NP-hard.*

The proof of the theorem is based on reducing 3-PARTITION to the problem $1|p_j = a_j \text{ or } a_j + b_j, h_j| \sum w_j T_j$. For a positive integer $t \in \mathbb{N}$, let integers $a_j \in \mathbb{N}, j \in \mathbb{N}_{3t}$, and $b \in \mathbb{N}$ such that $\frac{b}{4} < a_j < \frac{b}{2}, j \in \mathbb{N}_{3t}$ and $\sum_{j \in \mathbb{N}_{3t}} a_j = tb$. The reduction is based on the following transformation. Let the number of jobs $n = 4t - 1$. Let the partition jobs be such that for $j \in \mathbb{N}_{3t}$,

$$d_j = 0, \quad h_j = tb + 2(t - 1), \quad b_j = 1, \quad w_j = a_j - \frac{b}{4} \quad (3.8)$$

and

$$p_j = \begin{cases} a_j, & s_j \leq h_j, \\ a_j + b_j, & \text{otherwise.} \end{cases} \quad (3.9)$$

Introduce the notation $\mathbb{N}_{m,n} := \{m, m + 1, \dots, n\}$. Let the $t - 1$ enforcer jobs be such that for $j \in \mathbb{N}_{3t+1, 4t-1}$,

$$d_j = (j - 3t)(b + 1), \quad h_j = d_j - 1, \quad b_j = 1, \quad w_j = (b + b^2)(t^2 - t) \quad (3.10)$$

and

$$p_j = \begin{cases} 1, & s_j \leq h_j, \\ 1 + b_j, & \text{otherwise.} \end{cases} \quad (3.11)$$

The first $3t$ partition jobs are due at time 0, and the last $(t - 1)$ enforcer jobs are due at $b + 1, 2b + 2, \dots$ and so on. The deterioration dates of all partition jobs are set at $h_j = tb + 2(t - 1), j \in \mathbb{N}_{3t}$; while the deterioration dates of all enforcer jobs are set at $d_j - 1$, for $j \in \mathbb{N}_{3t+1, 4t-1}$, that is, one unit before their individual due dates, respectively. We introduce the set notation $\mathbb{S}_i := \{3i - 2, 3i - 1, 3i\}, i \in \mathbb{N}_t$ for sets of three partition jobs. In general, assume that for $i \in \mathbb{N}_t$,

$$a_{3i-2} + a_{3i-1} + a_{3i} = b + \delta_i, \quad (3.12)$$

where, due to $\frac{b}{4} < a_j < \frac{b}{2}, j \in \mathbb{N}_{3t}$, we must have

$$-\frac{b}{4} < \delta_i < \frac{b}{2}. \quad (3.13)$$

The quantity $\delta_i, i \in \mathbb{N}_t$ describes the difference from b of the sum of basic processing times of jobs in \mathbb{S}_i , $i \in \mathbb{N}_t$. Since $\sum_{j \in \mathbb{N}_{3t}} a_j = tb$, we deduce that

$$\sum_{i \in \mathbb{N}_t} \delta_i = 0. \quad (3.14)$$

For convenience we define $\Delta_0 = 0$ and for $i \in \mathbb{N}_t$,

$$\Delta_i = \sum_{j \in \mathbb{N}_i} \delta_j.$$

That is, Δ_i is the accumulative difference from ib of the cumulative basic processing times of the partition jobs up to the i -th set \mathbb{S}_i of 3 partition jobs. Note by (3.14), $\Delta_t = 0$. The following observation is useful.

Lemma 1

$$\sum_{j \in \mathbb{N}_t} j \delta_j = - \sum_{j \in \mathbb{N}_{t-1}} \Delta_j = - \sum_{j \in \mathbb{N}_t} \Delta_j. \quad (3.15)$$

Proof The proof can be done by direct calculation. The second equality is obvious because $\Delta_t = 0$. For the first equality, we have

$$\begin{aligned} \sum_{j \in \mathbb{N}_t} j\delta_j &= \delta_1 + 2\delta_2 + \dots + t\delta_t \\ &= (\delta_1 + \delta_2 + \dots + \delta_t) + (\delta_2 + \delta_3 + \dots + \delta_t) \\ &\quad + \dots + (\delta_{t-1} + \delta_t) + \delta_t \\ &= 0 - \delta_1 - (\delta_1 + \delta_2) - \dots \\ &\quad - (\delta_1 + \delta_1 + \dots + \delta_{t-2}) \\ &\quad - (\delta_1 + \delta_2 + \dots + \delta_{t-1}) \end{aligned}$$

where in the last equality we have used the equality $\sum_{j \in \mathbb{N}_t} \delta_j = 0$. Thus we continue by using the definition of Δ_j , $j \in \mathbb{N}_t$ to have

$$\begin{aligned} \sum_{j \in \mathbb{N}_t} j\delta_j &= -\Delta_1 - \Delta_2 - \dots - \Delta_{t-2} - \Delta_{t-1} \\ &= - \sum_{j \in \mathbb{N}_{t-1}} \Delta_j. \end{aligned}$$

The lemma is proved.

Let $\lceil x \rceil$ be the least integer greater than or equal to x , and for $k \in \mathbb{N}$, define the index set $J_k := \{3 \lceil \frac{k}{3} \rceil - 2, 3 \lceil \frac{k}{3} \rceil - 1, \dots, k\}$.

We are ready to prove the theorem by showing that there exists a schedule with an objective value

$$z^* = \frac{(t^2 - t)}{8}(b + b^2) + \sum_{k \in \mathbb{N}_{3t}} \sum_{j \in J_k} a_j \left(a_k - \frac{b}{4} \right) \quad (3.16)$$

if and only if there exists a solution for the 3-PARTITION problem.

Proof (Proof of Theorem 1)

If the 3-PARTITION problem has a solution, the corresponding $3t$ jobs thus can be partitioned into t subsets \mathbb{S}_i , $i \in \mathbb{N}_t$, of three jobs each, with the sum of the three processing times in each subset equal to b , that is, $\delta_i = 0$, for $i \in \mathbb{N}_t$, and the last $t - 1$ jobs are processed exactly during the intervals

$$[b, b+1], [2b+1, 2b+2], \dots, [(t-1)b+t-2, (t-1)b+t-1].$$

In this scenario, all the $t - 1$ enforcer jobs are not tardy and all the $3t$ partition jobs are tardy. The tardiness of each partition job equals to its completion time. Moreover, no job is deteriorated.

Let c_j , $j \in \mathbb{N}_{4t}$ be the completion time of each job. Let S_i be the starting time of the first job in each set \mathbb{S}_i , $i \in \mathbb{N}_t$. When no job is deteriorated, that is all jobs are processed with basic processing time, the completion time of each job j in \mathbb{S}_i , $i \in \mathbb{N}_t$ is given by

$$c_j = S_i + \sum_{k \in J_j} a_k.$$

The total weighted tardiness of the 3 jobs in \mathbb{S}_i , $i \in \mathbb{N}_t$, equals to

$$\begin{aligned} z_i &= \sum_{j \in \mathbb{S}_i} c_j w_j \\ &= S_i \sum_{j \in \mathbb{S}_i} w_j + \sum_{j \in \mathbb{S}_i} \sum_{k \in J_j} a_k w_j \\ &= [(i-1)b + (i-1)] \frac{b}{4} + \sum_{j \in \mathbb{S}_i} \sum_{k \in J_j} a_k \left(a_j - \frac{b}{4} \right), \end{aligned} \quad (3.17)$$

since $S_i = (i-1)b + (i-1)$ and $\sum_{j \in \mathbb{S}_i} w_j = \frac{b}{4}$, for each $i \in \mathbb{N}_t$. Thus the total weighted tardiness is $\sum_{i \in \mathbb{N}_t} z_i$ which sums to z^* given by (3.16).

Conversely, if such a 3-partition is not possible, there is at least one $\delta_i \neq 0$, $i \in \mathbb{N}_t$. We next argue that this must imply $\Delta z := z - z^* > 0$.

If $\Delta_i > 0$, $i \in \mathbb{N}_{t-1}$, then the i -th enforcer job will deteriorate to be processed in the extended time and entail a weighted tardiness. Introduce the notation $x_+ := \max\{x, 0\}$, and let $\mathcal{N}(i)$, $i \in \mathbb{N}_{t-1}$ denote the number of times the value $\Delta_j > 0$, $j \in \mathbb{N}_i$. For convenience, we define $\mathcal{N}(0) = 0$. The value $\mathcal{N}(i)$, with $0 \leq \mathcal{N}(i) \leq i$, coincides with the cumulative extended processing time of all the enforcer jobs up to the i -th enforcer job. This implies that the weighted tardiness of the i -th enforcer job is given by

$$((\Delta_i)_+ + \mathcal{N}(i)) (b + b^2)(t^2 - t).$$

On the other hand, our configuration of the deterioration dates for all the partition jobs ensures that no partition jobs can deteriorate. Therefore, by recalling equation (3.12), we have in this case that for $i \in \mathbb{N}_t$,

$$S_i = (i-1)b + (i-1) + \Delta_{i-1} + \mathcal{N}(i-1)$$

and

$$\sum_{j \in \mathbb{S}_i} w_j = \frac{b}{4} + \delta_i.$$

In view of equation (3.17) and the weighted tardiness of the enforcer job, we deduce that the change in the objective function value caused by δ_i , $i \in \mathbb{N}_t$, is given by

$$\begin{aligned} \Delta z_i &= ((i-1)b + (i-1) + \Delta_{i-1} + \mathcal{N}(i-1)) \delta_i \\ &\quad + \frac{b}{4} (\Delta_{i-1} + \mathcal{N}(i-1)) \\ &\quad + ((\Delta_i)_+ + \mathcal{N}(i)) (b + b^2)(t^2 - t). \end{aligned} \quad (3.18)$$

Therefore the total change in the objective function value is given by

$$\begin{aligned}
\Delta z &= \sum_{i \in \mathbb{N}_t} \Delta z_i \\
&\geq (1+b) \sum_{i \in \mathbb{N}_t} (i-1)\delta_i + \sum_{i \in \mathbb{N}_t} \Delta_{i-1} \left(\delta_i + \frac{b}{4} \right) \\
&\quad + (b+b^2)(t^2-t) \sum_{i \in \mathbb{N}_t} \mathcal{N}(i) \\
&= (1+b) \sum_{i \in \mathbb{N}_t} i\delta_i + \sum_{i \in \mathbb{N}_{t-1}} \Delta_i \left(\delta_{i+1} + \frac{b}{4} \right) \\
&\quad + (b+b^2)(t^2-t) \sum_{i \in \mathbb{N}_{t-1}} \mathcal{N}(i).
\end{aligned}$$

In the last equality again we have used $\Delta_t = \sum_{j \in \mathbb{N}_t} \delta_j = 0$. Now by equation (3.15), we continue to have

$$\begin{aligned}
\Delta z &\geq (1+b) \sum_{i \in \mathbb{N}_{t-1}} (-\Delta_i) + \sum_{i \in \mathbb{N}_{t-1}} \Delta_i \left(\delta_{i+1} + \frac{b}{4} \right) \\
&\quad + (b+b^2)(t^2-t) \sum_{i \in \mathbb{N}_{t-1}} \mathcal{N}(i) \\
&= \sum_{i \in \mathbb{N}_{t-1}} (-\Delta_i) \left(1 + \frac{3}{4}b - \delta_{i+1} \right) + \\
&\quad (b+b^2)(t^2-t) \sum_{i \in \mathbb{N}_{t-1}} \mathcal{N}(i).
\end{aligned} \tag{3.19}$$

If there is a $\Delta_i > 0$ for some $i \in \mathbb{N}_t$, then $\sum_{i \in \mathbb{N}_{t-1}} \mathcal{N}(i) \geq 1$. Recalling that $\frac{b}{2} > \delta_i > -\frac{b}{4}$, for all $i \in \Delta_t$, we have

$$\begin{aligned}
\sum_{i \in \mathbb{N}_{t-1}} (-\Delta_i) \left(1 + \frac{3}{4}b - \delta_{i+1} \right) &> (1+b) \sum_{i \in \mathbb{N}_{t-1}} (-\Delta_i) \\
&> (1+b) \sum_{i \in \mathbb{N}_{t-1}} \left(-i\frac{b}{2} \right) \\
&= -\frac{1}{4}b(1+b)t(t-1).
\end{aligned}$$

Thus in this case, $\Delta z > 0$ by equation (3.19). On the other hand, if all $\Delta_i \leq 0$, and at least one $\delta_i \neq 0$, $i \in \mathbb{N}_t$, then there must be at least one $\Delta_i < 0$. Thus in this case

$$\begin{aligned}
\Delta z &= \sum_{i \in \mathbb{N}_{t-1}} (-\Delta_i) \left(1 + \frac{3}{4}b - \delta_{i+1} \right) \\
&> 0
\end{aligned}$$

because $(1 + \frac{3}{4}b - \delta_{i+1}) > 0$ for $i \in \mathbb{N}_t$. Therefore $\Delta z = 0$ if and only if $\delta_i = 0$, $i \in \mathbb{N}_t$. We have proved the theorem.

Remark: with small changes, the above proof also shows that the scheduling problem of total weighted tardiness

(without deteriorating jobs), represented by $1 || \sum w_j T_j$, is strongly NP-hard, of which proofs might be found in [21, 23] and [29, p. 58].

After a reflection, we mention that the following three problems also are strongly NP-hard: 1) the problem of total weighted tardiness with deterioration jobs and job release times r_j ; 2) the problem of total tardiness with deterioration jobs and job release times r_j ; 3) the problem of maximum lateness of a single machine scheduling problem with job release time r_j . Recall that the maximum lateness is defined to be

$$L_{\max} = \max\{L_j : L_j = c_j - d_j, j \in \mathbb{N}_n\}.$$

Their proofs can be obtained by slightly modifying our previous proof. In fact, the assumption of release times makes the proof a lot of easier. We summarize these results in the following corollaries.

Corollary 2 *The problem $1|p_j = a_j \text{ or } a_j + b_j, h_j, r_j | \sum w_j T_j$ is strongly NP-hard.*

Corollary 3 *The problem $1|p_j = a_j \text{ or } a_j + b_j, h_j, r_j | \sum T_j$ is strongly NP-hard.*

Corollary 4 *The problem $1|p_j = a_j \text{ or } a_j + b_j, h_j, r_j | L_{\max}$ is strongly NP-hard.*

4 Heuristic algorithms

The problem under study is proved strongly NP-hard earlier, then some dispatching heuristics are needed to develop for solving the problem. In this section, the details of these heuristics are discussed. Dispatch heuristics gradually form the whole schedule by adding one job at a time with the best priority index among the unscheduled jobs. There are several existing heuristics designed for the problem without deteriorating jobs. Since the processing times of all jobs considered in our problem depend, respectively, on their starting times, these dispatching heuristics are modified for considering the characteristic of the problem.

Before introducing these procedures, the following notations are defined. Let N^s denote the *ordered set* of already-scheduled jobs and N^u the *unordered set* of unscheduled jobs. Hence $\mathbb{N}_n = N^s \cup N^u$ when the ordering of N^s is not in consideration. Let t^s denote the current time, i.e., the maximum completion time of the scheduled jobs in the set N^s . We shall call t^s the current time of the sequence N^s . Simultaneously, t^s is also the starting time of the next selected job. For each of the unscheduled jobs in N^u , its actual processing time is calculated based on its deteriorating date and the current time t^s .

For most scheduling problems with due dates, the *earliest due date* (EDD) rule is simple and efficient. In this paper, the rule is adopted to obtain a schedule by sorting jobs in non-decreasing order of their due dates. In the same way, the *weighted shortest processing time* (WSPT) schedules jobs in decreasing order of w_j/p_j . At each iteration, the actual processing times of unscheduled jobs in N^u are needed to recalculate. This is because when step-deteriorating effect is considered, the processing time of a job is variable. Even when all jobs are necessarily tardy, the WSPT rule does not guarantee an optimal schedule. Moreover, the weighted EDD (WEDD) rule introduced by Kanet and Li [17] sequences jobs in non-decreasing order of WEDD, where

$$\text{WEDD}_j = d_j/w_j, \quad (4.20)$$

The apparent tardiness cost (ATC) heuristic introduced by Vepsalainen and Morton [34] was developed for the total weighted tardiness problem when the processing time of a job is constant and known in advance. It showed relatively good performance compared with the EDD and the WSPT. The job with the largest ATC value is selected to be processed. The ATC for job j is determined by the following equation.

$$\text{ATC}_j = \frac{w_j}{p_j} \exp(-\max\{0, d_j - p_j - t^s\}/(\kappa\rho)), \quad (4.21)$$

where, κ is a “look-ahead” parameter usually between 0.5 and 4.5, and ρ is the average processing time of the rest of unscheduled jobs. The processing time of an already scheduled job $j \in N^s$ may be a_j or $a_j + b_j$ dependent on if it is deteriorated. Subsequently, the current time t_s is calculated upon the completion of the last job in N^s . The parameter ρ is calculated by averaging the processing times of unscheduled jobs in N^u assuming their starting time is at t^s .

Based on the cost over time (COVERT) rule [10] and the apparent urgency (AU) rule [26], Alidaee and Ramakrishnan [1] developed a class of heuristic named COVERT-AU for the standard single machine scheduling weighted tardiness problem. The COVERT-AU heuristic combines the two well known methods, i.e. COVERT and AU. At the time t^s , the COVERT-AU chooses the next job with the largest priority index CA_j calculated by the equation

$$\text{CA}_j = \frac{w_j}{p_j}(\kappa\rho/(\kappa\rho + \max\{0, d_j - p_j - t^s\})). \quad (4.22)$$

For the convenience of description, the heuristic with equation (4.22) is denoted by CA in this paper hereafter.

The *weighted modified due date* (WMDD) rule was developed by Kanet and Li [17] based on modified due

date (MDD). In this method, the jobs are processed in non-decreasing order of WMDD. The WMDD is calculated by the equation

$$\text{WMDD}_j = \frac{1}{w_j}(\max\{p_j, (d_j - t^s)\}). \quad (4.23)$$

Note that when all job weights are equal, the WMDD is equal to the MDD.

The above heuristics need to recalculate the processing time of the next job for obtaining the priority index except for the EDD and the WEDD. The procedures to recalculate the processing time of the next job is significantly different from those for the problem without step-deterioration. In order to illustrate how these heuristics work, the detailed steps of the WMDD, as an example, are shown in Algorithm 1. For other heuristics, the only difference is the calculation of the priority index.

Algorithm 1 The WMDD

- 1: Input the initial data of a given instance;
 - 2: Set $N^s = []$, $t^s = 0$ and $N^u = \{1, 2, \dots, n\}$;
 - 3: Set $k = 1$;
 - 4: **repeat**
 - 5: Compute the processing time of each job in the set N^u based on the current time t^s ;
 - 6: Calculate the WMDD value of each job in N^u according to equation (4.23);
 - 7: Choose job j from the set N^u with the smallest value WMDD_j to be scheduled in the k th position;
 - 8: Update the tardiness of job j : $T_j = \max\{t^s + p_j - d_j, 0\}$, and $N^s = [N^s, j]$;
 - 9: Delete job j from N^u ;
 - 10: $k = k + 1$;
 - 11: **until** the set N^u is empty
 - 12: Calculate the total weighted tardiness of the obtained sequence N^s .
-

A very effective and simple combination search heuristic for minimizing total tardiness was proposed by Guo et al [13]. The heuristic is called “Simple Weighted Search Procedure” (SWSP) and works as follows: a combined value of parameters a_j , p_j and h_j for job j is calculated as

$$m_j = \gamma_1 d_j + \gamma_2 p_j + \gamma_3 h_j, \quad (4.24)$$

where γ_1 , γ_2 and γ_3 are three positive constants. In the SWSP, jobs are sequenced in non-decreasing order of m -value. To accommodate the case of the weighted tardiness, equation (4.24) is modified to compute a priority index m' for a job j calculated by the equation

$$m'_j = \frac{1}{w_j}(\gamma_1 d_j + \gamma_2 p_j + \gamma_3 h_j). \quad (4.25)$$

The modified method is called "Modified Simple Weighted Search Procedure" (MSWSP). In equation (4.25), the values of γ_1 , γ_2 and γ_3 are determined by using a dynamically updating strategy. The updating strategy is similar to that proposed by Guo et al [13]. Specifically, parameter γ_1 is linearly increased by 0.1 at each iteration and its range is varied from $\gamma_{1\min}$ to $\gamma_{1\max}$. In this study, $\gamma_{1\min} = 0.2$ and $\gamma_{1\max} = 0.9$ are chosen based on preliminary tests by using randomly generated instances. The parameter γ_2 adopts a similar approach with $\gamma_{1\min}$ and $\gamma_{1\max}$ replaced by $\gamma_{2\min} = 0.1$ and $\gamma_{2\max} = 0.7$, respectively. Once the values of parameters γ_1 and γ_2 are determined, the parameter $\gamma_3 := \max\{1 - \gamma_1 - \gamma_2, 0.1\}$. The detailed steps of the MSWSP is shown in Algorithm 2.

Algorithm 2 The MSWSP

```

1: Input the initial data of a given instance;
2: Set  $c_{[0]} = 0$ ,  $N^u = \{1, \dots, n\}$  and  $N^s = []$ ;
3: Generate the entire set  $\Omega$  of possible triples of weights.
   For each triple  $(\omega_1, \omega_2, \omega_3) \in \Omega$ , perform the following
   steps;
4: Choose job  $i$  with minimal due date to be scheduled in
   the first position;
5:  $c_{[1]} = c_{[0]} + a_i$ ,  $N^s = [N^s, i]$ ;
6: delete job  $i$  from  $N^u$ ;
7: set  $k = 2$ ;
8: repeat
9:   choose job  $j$  from  $N^u$  with the smallest value  $m'_j$  to
   be scheduled in the  $k$ th position;
10:  if  $c_{[k-1]} > h_j$  then
11:     $c_{[k]} = c_{[k-1]} + a_j + b_j$ ;
12:  else
13:     $c_{[k]} = c_{[k-1]} + a_j$ ;
14:  end if
15:   $N^s = [N^s, j]$ ;
16:  delete job  $j$  from  $N^u$ ;
17: until the set  $N^u$  is empty.
18: Calculate the total weighted tardiness of the obtained
   schedule  $N^s$ ;
19: Output the final solution  $N^s$ .
```

In order to further improve the quality of the near-optimal solutions, a pairwise swap movement (PS) is incorporated into these heuristics. Let N^s be the sequence output by a heuristic. A swap operation chooses a pair of jobs in positions i and j , $1 \leq i, j \leq n$, from the sequence N^s , and exchanges their positions. Denote the new sequence by N_{ji}^s . Subsequently, the total weighted tardiness of the sequence N_{ji}^s is calculated. If the new sequence N_{ji}^s is better with a smaller tardiness than the incumbent one, the incumbent one is replaced by the new sequence. The swap operation is repeated for any combination of two indices i and j , where $1 \leq i < j \leq n$. Thus the size of the pairwise swap movement (PS) is $n(n-1)/2$. In the following, a

heuristic with the PS movement is denoted by the symbol ALG_{PS} , where ALG is one of the above mentioned heuristic algorithms. For example, EDD_{PS} represents that the earliest due date rule is applied first, then the solution obtained by the EDD is further improved by the PS movement.

5 Computational experiments

In this section, the computational experiments and results are presented to analyze the performance of the above dispatching heuristics. Firstly, randomly generated test problem instances varying from small to large sizes are described. Next, preliminary experiments are carried out to determine appropriate values for the parameters used in some of the heuristics. Then, a comparative analysis of all seven dispatching heuristics is performed. Furthermore, the results of the best method are compared with optimal solutions delivered by ILOG CPLEX 12.5 for small-sized problem instances. All heuristics were coded in MATLAB 2010 and run on a personal computer with Pentium Dual-Core E5300 2.6 GHz processor and 2 GB of RAM.

5.1 Experimental design

The problem instances were generated using the method proposed by Guo et al [13] as follows. For each job j , a basic processing time a_j was generated from the uniform distribution $[1, 100]$, a weight w_j was generated from the uniform distribution $[1, 10]$, and a deteriorating penalty b_j is generated from the uniform distribution $[1, 100 \times \tau]$, where $\tau = 0.5$. Problem hardness is likely to depend on the value ranges of deteriorating dates and due dates. For each job j , a deteriorating date h_j was drawn from the uniform distribution over three intervals $H_1 := [1, A/2]$, $H_2 := [A/2, A]$ and $H_3 := [1, A]$, where $A = \sum_{j \in \mathbb{N}_n} a_j$. Meanwhile, a due date d_j was generated from the uniform distribution $[C'_{\max}(1 - T - R/2), C'_{\max}(1 - T + R/2)]$, where C'_{\max} is the value of the maximum completion time obtained by scheduling the jobs in the non-decreasing order of the ratios a_j/b_j , $j \in \mathbb{N}_n$, T is the average tardiness factor and R is the relative range of due dates. Both T and R were set at 0.2, 0.4, 0.6, 0.8 and 1.0.

Overall, 75 different combinations were generated for different h , T and R . For the purpose of obtaining optimal solutions, the number of jobs in each instance was taken to be one of the two sizes of 8 and 10. For the heuristics, the number of jobs can be varied from the small sizes to the large sizes, that comprises 14 sizes including 8, 10, 15, 20, 25, 30, 40, 50, 75,

100, 250, 500, 750 and 1000. In each combination of h , T , R , and n , 10 replicates were generated and solved. Thus, there are 750 instances for each problem size, totalling 10500 problem instances, which are available from http://www.researchgate.net/profile/Peng_Guo9.

In general, the performances of a heuristic is measured by the average relative percentage deviation (RPD) of the heuristic solution values from optimal solutions value or best solution values. The average RPD value is calculated as $\frac{1}{K} \sum_{k=1}^K \frac{Z_{\text{alg}}^k - Z_{\text{opt}}^k}{Z_{\text{opt}}^k} \times 100$, where K is the number of problem instances and Z_{alg}^k and Z_{opt}^k are the objective function value of the heuristic method and the optimal solution value for instance k , respectively.

The objective function value delivered by a heuristics may be equal to 0 for an instance with a low tardiness factor T and a high relative range R of due dates. The zero objective value means that all jobs are finished on time. It is troublesome to obtain the RPD in this case, since necessarily $Z_{\text{opt}}^k = 0$, thus it leads to a division by 0 that is undefined. To avoid this situation, in this paper, the relative improvement versus the worst result (RIVW) used by Valente and Schaller [33] is adopted to evaluate the performance of a proposed heuristics. For a given instance, the RIVW for a heuristic is defined by the following way. Let Z_{best} and Z_{worst} denote the best and worst solution values delivered by all considered heuristics in comparison, respectively. When $Z_{\text{best}} = Z_{\text{worst}}$, the RIVW value of a heuristic algorithm is set to 0. Otherwise, the RIVW value is calculated as

$$\text{RIVW} = (Z_{\text{worst}} - Z_{\text{alg}}) / Z_{\text{worst}} \times 100.$$

Based on the definition of RIVW, it can be observed that the bigger the RIVW value, the better the quality of the corresponding solution.

5.2 Parameter selection

In order to select an appropriate value for the parameter κ , preliminary tests were conducted on a separate problem set, which contains instances with 20, 50, 100, and 500 jobs. For each of these job sizes n , 5 replicates were produced for each combination of n , h , T and R . Subsequently, for each problem instance, the objective function value was obtained by using all considered seven heuristics with a candidate value of κ . Then the results of all problem instances were analyzed to determine the best value of κ . The candidate values of the parameter κ are chosen to be 0.5, 1.0, ..., 4.5, which are usually used in the ATC and the CA for traditional single machine problem [34, 1]. The computational tests show that the solutions delivered by the

ATC and the CA with $\kappa = 0.5$ are relatively better compared with the results obtained by other values of κ . Thus, κ is set to 0.5 in the sequel.

5.3 Experimental results

The computational results of the proposed seven heuristics are listed in Table 5.1. Specifically, this table provides the mean RIVW values for each heuristic, as well as the number of instances with the best solution (Num_{best}) found by a heuristic method from 750 instances for each problem size. Each mean RIVW value for a heuristic and a particular problem size in Table 5.1 is the average of RIVW values from the 750 instances for a given problem size. From the table, the EDD and WEDD rules are clearly outperformed by the WSPT, ATC, CA, WMDD, and MSWSP heuristics. The mean RIVW values delivered by EDD and WEDD are significantly less than that given by other heuristics. This is due to the fact that the EDD rule only considers job due dates, while the WEDD relies only on weights and due dates. In addition, the two rules do not consider the effect of step-deterioration in calculating the priority index. It is worthwhile to note that the results achieved by the WEDD is better than that given by the EDD.

As far as the remaining methods, the RIVW values obtained by the WSPT and MSWSP heuristics are worse than that of other three (ATC, CA and WMDD) heuristics. But the performance of the MSWSP is better than the WSPT. This indicates that the MSWSP can produce good results for our problem, but it fails to obtain better solutions compared with the three improved methods (ATC, CA and WMDD).

There is no significant difference between the results produced by the ATC, CA and WMDD heuristics. The CA procedure provides slightly higher mean relative improvement versus the worst result values. For large-sized instances, the number of the best solutions achieved by the CA is much higher than the ATC and the WMDD. Therefore, *the CA procedure can be deemed as the best one among the seven dispatching heuristic algorithms.*

Computational times of these dispatching heuristic algorithms for each job size are listed in Table 5.2. As the size of instances increases, the CPU time of all methods grows at different degrees. Totally, the MSWSP consumes the most time compared with other algorithms, but surprisingly its maximum CPU time is only 5.71 seconds for the intractable instance with 1000 jobs. The average CPU time of the CA which is 0.66 seconds is less than that of the MSWSP. Since the EDD and the WEDD mainly depend on the ranking index of all jobs' due dates, their computational times are less than

Table 5.1 Computational results of the dispatching heuristic algorithms

<i>n</i>	RIVW(%)							Num _{best}						
	EDD	WSPT	WEDD	ATC	CA	WMDD	MSWSP	EDD	WSPT	WEDD	ATC	CA	WMDD	MSWSP
8	15.56	39.87	28.63	48.50	55.01	49.06	50.40	87	202	32	331	375	307	206
10	15.38	40.76	26.66	49.31	57.68	52.36	50.57	70	149	10	299	367	278	145
15	16.97	40.15	28.23	55.60	62.25	57.82	53.01	78	80	4	274	340	249	104
20	17.89	41.35	27.50	57.73	65.56	60.42	54.13	70	72	2	237	363	232	79
25	16.92	41.11	27.60	58.49	67.46	62.92	53.26	67	55	2	206	373	241	72
30	17.39	41.47	27.41	59.93	68.64	64.49	52.52	66	61	0	190	388	215	68
40	18.44	41.10	26.38	61.81	70.31	66.22	52.84	77	46	1	166	383	242	78
50	18.76	40.59	27.31	62.62	71.22	67.17	52.30	71	30	0	183	400	237	71
75	19.00	41.34	27.41	63.65	72.03	68.59	51.70	71	31	0	165	421	247	71
100	20.13	41.07	27.73	64.49	72.81	69.09	51.86	75	36	0	143	428	254	75
250	19.54	41.37	27.46	65.91	73.74	69.47	49.92	78	42	0	124	472	250	78
500	19.55	41.47	27.42	66.47	74.01	69.42	49.29	68	34	0	116	487	245	68
750	19.46	41.54	27.37	66.44	74.08	69.30	49.12	69	35	0	105	499	243	69
1000	19.50	41.46	27.32	66.38	74.13	69.32	48.92	67	32		95	502	250	67
Total								1014	905	51	2634	5798	3490	1251
Avg.	18.18	41.05	27.46	60.52	68.50	63.97	51.42	72	65	4	188	414	249	89

Table 5.2 Computational times of dispatching heuristic algorithms

n	CPU Time(s)						
	EDD	WSPT	WEDD	ATC	CA	WMDD	MSWSP
8	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.01
10	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.01
15	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.02
20	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.03
25	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.04
30	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.05
40	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.06
50	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0.08
75	<0.01	0.01	<0.01	0.01	0.02	0.01	0.14
100	<0.01	0.02	<0.01	0.02	0.05	0.02	0.20
250	<0.01	0.12	<0.01	0.14	0.30	0.14	0.65
500	<0.01	0.47	<0.01	0.58	1.19	0.56	1.81
750	<0.01	1.11	<0.01	1.33	2.72	1.31	3.48
1000	<0.01	2.05	<0.01	2.42	4.91	2.39	5.67
Avg.	<0.01	0.27	<0.01	0.32	0.66	0.32	0.88

0.01 seconds. It was observed that the CPU time of the other five algorithms follow almost the same trend.

Subsequently, the solutions delivered by the seven dispatching heuristics are improved by the PS movement. The seven heuristics with the PS movement are denoted by EDD_{PS} , $WSPT_{PS}$, $WEDD_{PS}$, ATC_{PS} , CA_{PS} , $WMDD_{PS}$ and $MSWSP_{PS}$, respectively. A comparison of these methods is given in Table 5.3. Again, this table lists the mean relative improvement versus the worst result (RIVW) for each algorithm and the number of instances with the best solution found by each of the seven algorithms with the PS movement.

From Table 5.3, it can be observed that the CA_{PS} provides the best performance among these procedures. In fact, the CA_{PS} not only gives the largest RIVW value, but also gives a better solution for most of the instances. For medium- and large-sized instances, the CA_{PS} shows better performance in terms of the number of best solution (Num_{best}) compared with the other six methods. In particular, for the case with 1000 jobs, the CA_{PS} gives the best solutions for 538 over the 750 instances. It is found that CA_{PS} delivers best solutions for on average 473 out of 750 instances for all job sizes. The average RIVW values delivered by the $WEDD_{PS}$, ATC_{PS} , CA_{PS} , $WMDD_{PS}$ and $MSWSP_{PS}$ are more than 40%. The RIVW value of the $WSPT$ is only 15.86%, significantly less than that achieved by the other six methods.

Computational times of these methods are listed in Table 5.4. The average computational times of the seven methods are very close, and the gap of the average CPU times between these methods is not more than one second. As expected, the CPU times of these algorithms are increased as the number of jobs increases. But the computational times of the seven methods are not more than 80 seconds even for the 1000-job case.

In order to further analyze the results, the one-way Analysis of Variance (ANOVA) is used to check whether the observed difference in the RIVW values for the dispatching heuristics with the PS movement are statistically significant. The $WSPT_{PS}$ is removed from the statistical analysis since it is clearly worse than the remaining ones. The means plot and the Fisher Least Significant Difference (LSD) intervals at the 95% confidence level are shown in Figure 5.1. If the LSD intervals of two algorithms are not overlapped, the performances of the tested algorithms are statistically significantly different. Otherwise, the performances of the two algorithms do not lie significantly in the difference. As it can be seen, ATC_{PS} , CA_{PS} and $WMDD_{PS}$ are not statistically different because their confidence intervals are overlapped. This observation is really important since it gives the conclusion that can not be obtained

from a table of average RIVW results. Moreover, the CA_{PS} and $WMDD_{PS}$ are statistically significantly better than the other three methods (EDD_{PS} , $WEDD_{PS}$ and $MSWSP_{PS}$) by having their LSD intervals of the RIVW values higher than those of other methods. However, the $WEDD_{PS}$ and the $MSWSP_{PS}$ are not statistically different due to their overlapping confidence intervals.

To evaluate the effect of the PS movement, a comparison of the CA heuristic with the CA_{PS} procedure is provided in Table 5.5. Table 5.5 gives the mean relative improvements versus the worst result values for the two procedures, as well as the number of times the CA_{PS} procedure performs better than (Num_{better}) or equal the CA (Num_{equal}). Since the heuristic CA always gives worse result when comparing with the CA_{PS} , its RIVW values in Table 5.5 equal to zero for all instances. On average, the RIVW values obtained by the CA_{PS} are 24.71% better than the RIVW values achieved by the CA. These results show that *the PS movement can significantly improve the quality of solutions delivered by the CA heuristic for most of the instances.*

Table 5.5 Comparison of CA with CA_{PS}

n	RIVW(%)		CA_{PS} versus CA	
	CA	CA_{PS}	Num_{equal}	Num_{better}
8	0.00	18.15	225	525
10	0.00	19.56	171	579
15	0.00	24.22	80	670
20	0.00	25.73	59	691
25	0.00	26.08	44	706
30	0.00	26.09	47	703
40	0.00	27.92	46	704
50	0.00	26.74	55	695
75	0.00	27.63	66	684
100	0.00	28.09	72	678
250	0.00	25.07	103	647
500	0.00	24.40	110	640
750	0.00	23.45	117	633
1000	0.00	22.86	119	631
Avg.	0.00	24.71	94	656

Next, the CA_{PS} heuristic is compared with the optimal solutions delivered by the CPLEX 12.5 for instances with 8 jobs and 10 jobs. Here, the performance of the heuristic is measured by the mean relative improvement of the optimum objective function value versus the heuristic solution (RIVH), as well as the number of instances with the optimal solution given by the heuristic (Num_{opt}). For a given instance, the relative improvement of the optimum objective function value versus heuristic is calculated as follows. When $Z_{opt} = Z_{alg}$, the RIVH value is set to 0. Otherwise, the

Table 5.3 Computational results of dispatching heuristic algorithms with the PS movement

n	RIVW(%)							Num _{best}						
	EDD _{PS}	WSPT _{PS}	WEDD _{PS}	ATC _{PS}	CA _{PS}	WMDD _{PS}	MSWSP _{PS}	EDD _{PS}	WSPT _{PS}	WEDD _{PS}	ATC _{PS}	CA _{PS}	WMDD _{PS}	MSWSP _{PS}
8	26.95	12.84	29.26	31.67	33.12	31.90	32.54	243	441	288	506	535	514	422
10	28.76	14.37	32.27	35.18	36.57	35.79	35.02	162	339	179	460	483	468	335
15	33.13	15.04	36.71	40.43	42.39	40.40	39.80	148	233	140	392	435	389	218
20	35.00	15.81	38.67	43.41	44.91	43.64	42.11	139	202	110	370	416	370	181
25	35.88	14.58	40.10	44.61	46.77	44.86	42.66	131	166	98	346	426	358	160
30	36.92	14.06	41.30	46.29	47.81	46.46	43.76	134	181	113	333	423	351	158
40	37.79	16.21	42.93	48.17	50.03	48.53	45.07	135	174	104	318	420	365	134
50	37.99	16.08	43.40	48.96	50.79	49.45	45.68	130	154	98	305	441	320	133
75	38.47	16.16	44.23	50.22	52.24	50.65	46.34	137	164	100	272	460	304	139
100	38.97	15.78	44.96	51.10	53.00	51.70	46.68	136	153	101	264	471	291	133
250	38.98	16.94	45.44	51.99	54.21	52.49	47.02	141	159	101	234	527	254	140
500	38.87	17.72	45.43	52.20	54.56	52.66	46.95	141	161	103	244	520	263	140
750	38.58	18.45	45.36	51.88	54.51	52.50	46.82	144	166	100	252	523	254	143
1000	38.37	18.02	45.20	51.67	54.52	52.44	46.64	147	166	105	240	538	252	146
Total								2068	2859	1740	4536	6618	4753	2582
Avg.	36.05	15.86	41.09	46.27	48.24	46.68	43.36	148	204	124	324	473	340	184

Table 5.4 Computational times of dispatching heuristic algorithms with the PS movement

n	CPU Time(s)						
	EDD _{PS}	WSPT _{PS}	WEDD _{PS}	ATC _{PS}	CA _{PS}	WMDD _{PS}	MSWSP _{PS}
8	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.01
10	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.01
15	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.02
20	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01	0.03
25	<0.01	<0.01	<0.01	<0.01	0.01	<0.01	0.04
30	<0.01	0.01	<0.01	0.01	0.01	0.01	0.05
40	0.01	0.02	0.01	0.02	0.02	0.02	0.08
50	0.02	0.03	0.02	0.03	0.04	0.03	0.11
75	0.07	0.08	0.07	0.08	0.10	0.08	0.21
100	0.14	0.16	0.14	0.17	0.19	0.16	0.34
250	1.47	1.56	1.47	1.60	1.75	1.60	2.12
500	9.71	9.87	9.79	10.10	10.66	10.12	11.56
750	30.3	30.28	30.35	30.96	32.06	30.96	33.69
1000	69.43	68.93	69.61	70.29	72.10	70.27	74.75
Avg.	7.94	7.92	7.96	8.09	8.35	8.09	8.79

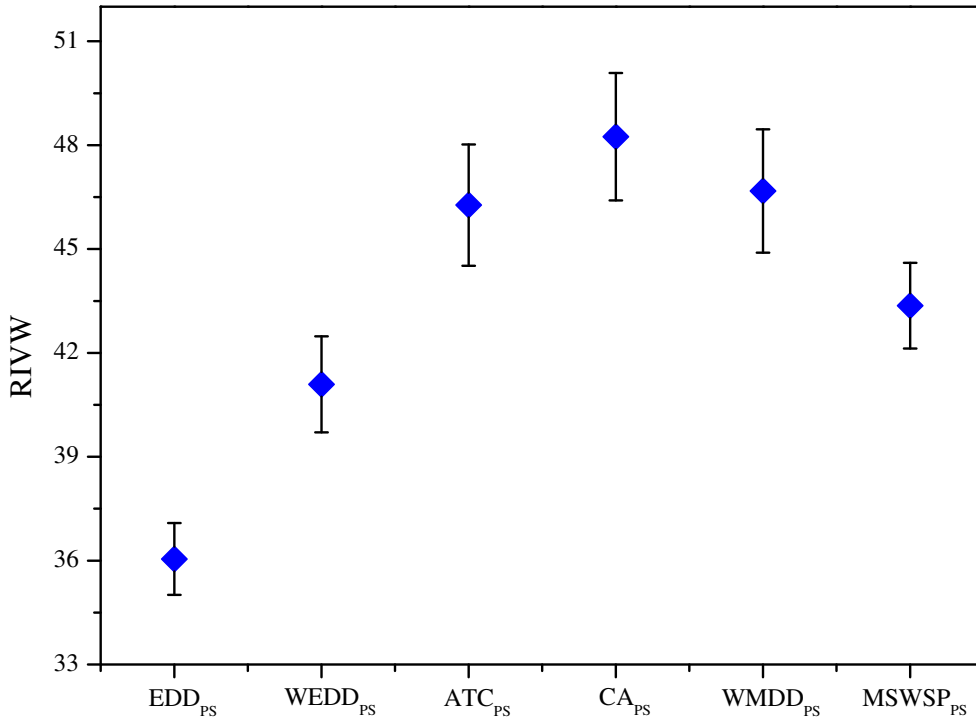


Fig. 5.1 Means plot and the LSD intervals (at the 95% confidence level) for the different dispatching heuristic algorithms with the PS movement

RIVH value is calculated as

$$\text{RIVH} = (Z_{\text{alg}} - Z_{\text{opt}}) / Z_{\text{alg}} \times 100.$$

According to the definition of the RIVH value, the smaller the mean RIVH value, the better the quality of the solutions delivered by a heuristic is for the set of instances.

The comparison results were shown in Table 5.6 and 5.7. The two tables show that *the tardiness factor T and the deteriorating interval H can significantly affect the performance of the heuristic CA_{PS}* . For large values of T , the RIVH values are relatively small and the corresponding objective function values are on average quite close to the optimum achieved by the CPLEX. When the deteriorating interval is H_2 , most jobs tend to have large deteriorating dates, and may not be deteriorated. Thus the RIVH values of the test instances with H_2 are less than that of the instances with H_1 and H_3 . This observation has been demonstrated by Cheng et al [8] for parallel machine scheduling problem. In this case, the heuristic CA_{PS} can give more optimal solutions compared with the instances with H_1 and H_3 . Overall, the CA_{PS} is effective in solving the problem under consideration since the maximum mean RIVH value is below 20% for instances with 10 jobs.

Table 5.6 Comparison with optimum results for instances with 8 jobs.

T	R	RIVH(%)			Num _{opt}		
		H_1	H_2	H_3	H_1	H_2	H_3
0.2	0.2	40.39	1.07	16.94	0	7	5
	0.4	48.40	0.47	34.66	3	8	4
	0.6	9.46	7.97	35.52	7	7	5
	0.8	18.78	10.00	0.00	7	9	10
	1	9.29	4.56	6.25	7	8	8
0.4	0.2	13.50	2.32	14.23	1	7	2
	0.4	7.00	0.00	7.50	3	10	3
	0.6	12.88	4.62	4.48	3	6	8
	0.8	33.70	7.99	14.46	4	6	5
	1	10.30	9.01	18.17	8	5	3
0.6	0.2	8.50	1.31	1.90	3	6	7
	0.4	9.94	0.98	8.97	2	6	4
	0.6	6.38	0.50	2.27	4	5	5
	0.8	5.84	2.55	10.09	5	7	1
	1	13.31	0.03	4.58	3	8	5
0.8	0.2	3.43	0.36	3.43	7	7	5
	0.4	9.58	0.14	1.09	3	8	6
	0.6	4.17	0.12	1.71	3	9	5
	0.8	3.32	0.87	0.48	4	7	7
	1	3.25	1.06	1.85	5	6	7
1	0.2	0.77	0.13	0.78	8	7	6
	0.4	1.16	0.00	2.35	6	9	3
	0.6	4.02	0.00	2.36	4	9	6
	0.8	2.79	0.00	1.48	4	9	6
	1	0.75	0.13	1.28	6	8	5
Avg.		11.24	2.25	7.87	4.40	7.36	5.24

Table 5.7 Comparison with optimum results for instances with 10 jobs

T	R	RIVH(%)			Num _{opt}		
		H_1	H_2	H_3	H_1	H_2	H_3
0.2	0.2	55.29	2.52	38.39	2	7	3
	0.4	70.00	20.40	30.71	3	5	5
	0.6	31.10	5.64	16.71	5	8	8
	0.8	25.00	16.07	17.36	7	8	5
	1	14.85	22.23	31.82	7	6	5
0.4	0.2	18.28	0.88	10.54	1	7	4
	0.4	20.75	2.59	27.79	1	5	0
	0.6	43.98	7.06	21.03	0	5	3
	0.8	35.02	16.17	23.44	2	3	3
	1	16.40	14.17	28.16	0	4	4
0.6	0.2	10.27	1.27	4.09	1	7	4
	0.4	7.45	5.21	5.36	5	3	5
	0.6	3.01	1.55	5.14	2	4	4
	0.8	7.70	3.44	5.92	2	5	3
	1	8.51	2.71	5.88	1	3	3
0.8	0.2	2.31	0.53	0.75	4	7	5
	0.4	8.01	0.03	3.61	1	8	3
	0.6	3.76	1.03	2.81	3	3	5
	0.8	3.52	0.29	2.05	5	7	2
	1	4.66	1.06	3.66	3	6	3
1	0.2	0.67	0.18	1.14	6	6	4
	0.4	2.66	0.20	1.61	2	7	4
	0.6	2.93	0.13	0.92	1	9	5
	0.8	1.44	0.01	0.92	4	9	5
	1	1.37	0.02	1.51	4	9	3
Avg.		15.96	5.01	11.65	2.88	6.04	3.92

6 Conclusions

The total weighted tardiness as the general case has more important meaning in the practical situation. In this paper, the single machine scheduling problem with step-deteriorating jobs for minimizing the total weighted tardiness was addressed. Based on the characteristics of this problem, a mathematical programming model is presented for obtaining the optimal solution, and, the proof of the strong NP-hardness for the problem under consideration is given. Afterwards, seven heuristics are designed to obtain the near-optimal solutions for randomly generated problem instances. Computational results show that these dispatching heuristics can deliver relatively good solutions at low cost of computational time. Among these dispatching heuristics, the CA procedure as the best solution method can quickly generate a good schedule even for large instances. Moreover, the test results clearly indicate that these methods can be significantly improved by the pairwise swap movement.

In the future, the consideration of developing meta-heuristics such as a genetic algorithm or ant colony optimization approach might be an interesting issue. For medium-sized problems, it is possible that a meta-heuristic could give better solutions within reasonable computational time. Another consideration is to inves-

tigate the total weighted tardiness problem with the step-deteriorating effects under other machine settings, such as parallel machines or flow-shops.

Acknowledgements This work is supported by the National Natural Science Foundation of China (No. 51405403) and the Fundamental Research Funds for the Central Universities (No. 2682014BR019).

References

1. Alidaee B, Ramakrishnan K (1996) A computational experiment of covert-au class of rules for single machine tardiness scheduling problem. *Computers & industrial engineering* 30(2):201–209
2. Baker KR, Trietsch D (2009) *Principles of Sequencing and Scheduling*. John Wiley & Sons
3. Barketau M, Cheng TE, Ng C, Kotov V, Kovalyov MY (2008) Batch scheduling of step deteriorating jobs. *Journal of Scheduling* 11(1):17–28
4. Bilge Ü, Kurtulan M, Kırac F (2007) A tabu search algorithm for the single machine total weighted tardiness problem. *European Journal of Operational Research* 176(3):1423–1435
5. Cheng T, Ding Q (2001) Single machine scheduling with step-deteriorating processing times. *European Journal of Operational Research* 134(3):623–630
6. Cheng T, Ding Q, Kovalyov MY, Bachman A, Janiak A (2003) Scheduling jobs with piecewise linear decreasing processing times. *Naval Research Logistics (NRL)* 50(6):531–554
7. Cheng TE, Ng C, Yuan J, Liu Z (2005) Single machine scheduling to minimize total weighted tardiness. *European Journal of Operational Research* 165(2):423–443
8. Cheng W, Guo P, Zhang Z, Zeng M, Liang J (2012) Variable neighborhood search for parallel machines scheduling problem with step deteriorating jobs. *Mathematical Problems in Engineering* 928312:1–20
9. Crauwels H, Potts CN, Van Wassenhove LN (1998) Local search heuristics for the single machine total weighted tardiness scheduling problem. *INFORMS Journal on computing* 10(3):341–350
10. Fisher ML (1976) A dual algorithm for the one-machine scheduling problem. *Mathematical programming* 11(1):229–251
11. Gawiejnowicz S (2008) *Time-dependent scheduling*. Springer
12. Graham RL, Lawler EL, Lenstra JK, Kan A (1979) Optimization and approximation in deterministic sequencing and scheduling: a survey. *Annals of Discrete Mathematics* 5:287–326

13. Guo P, Cheng W, Wang Y (2014) A general variable neighborhood search for single-machine total tardiness scheduling problem with step-deteriorating jobs. *Journal of Industrial and Management Optimization* 10(4):1071–1090, DOI 10.3934/jimo.2014.10.1071
14. He C, Wu C, Lee W (2009) Branch-and-bound and weight-combination search algorithms for the total completion time problem with step-deteriorating jobs. *Journal of the Operational Research Society* 60(12):1759–1766
15. Jeng A, Lin B (2004) Makespan minimization in single-machine scheduling with step-deterioration of processing times. *Journal of the Operational Research Society* 55(3):247–256
16. Jeng A, Lin B (2005) Minimizing the total completion time in single-machine scheduling with step-deteriorating jobs. *Computers & operations research* 32(3):521–536
17. Kanet JJ, Li X (2004) A weighted modified due date rule for sequencing to minimize weighted tardiness. *Journal of Scheduling* 7(4):261–276
18. Koulamas C (2010) The single-machine total tardiness scheduling problem: review and extensions. *European Journal of Operational Research* 202(1):1–7
19. Kubiak W, van de Velde S (1998) Scheduling deteriorating jobs to minimize makespan. *Naval Research Logistics (NRL)* 45(5):511–523
20. Kunnathur AS, Gupta SK (1990) Minimizing the makespan with late start penalties added to processing times in a single facility scheduling problem. *European Journal of Operational Research* 47(1):56–64
21. Lawler EL (1977) A 'pseudopolynomial' algorithm for sequencing jobs to minimize total tardiness. *Annals of discrete Mathematics* 1:331–342
22. Layegh J, Jolai F, Amalnik MS (2009) A memetic algorithm for minimizing the total weighted completion time on a single machine under step-deterioration. *Advances in Engineering Software* 40(10):1074–1077
23. Lenstra J, Kan AR, Brucker P (1977) Complexity of machine scheduling problems. *Annals of Discrete Mathematics* 1:343–362
24. Leung J, Ng C, Cheng T (2008) Minimizing sum of completion times for batch scheduling of jobs with deteriorating processing times. *European Journal of Operational Research* 187(3):1090–1099
25. Mor B, Mosheiov G (2012) Batch scheduling with step-deteriorating processing times to minimize flowtime. *Naval Research Logistics (NRL)* 59(8):587–600
26. Morton T, Rachamadugu R, Vepsalainen A (1984) Accurate myopic heuristics for tardiness scheduling. GSIA Working Paper No, 36-83-84, Carnegie-Mellon University, PA
27. Mosheiov G (1995) Scheduling jobs with step-deterioration; minimizing makespan on a single-and multi-machine. *Computers & industrial engineering* 28(4):869–879
28. Moslehi G, Jafari A (2010) Minimizing the number of tardy jobs under piecewise-linear deterioration. *Computers & Industrial Engineering* 59(4):573–584
29. Pinedo M (2012) *Scheduling: theory, algorithms, and systems*. Springer
30. Potts C, van Wassenhove LN (1991) Single machine tardiness sequencing heuristics. *IIE transactions* 23(4):346–354
31. Sen T, Sulek JM, Dileepan P (2003) Static scheduling research to minimize weighted and unweighted tardiness: a state-of-the-art survey. *International Journal of Production Economics* 83(1):1–12
32. Sundararaghavan P, Kunnathur A (1994) Single machine scheduling with start time dependent processing times: some solvable cases. *European Journal of Operational Research* 78(3):394–403
33. Valente J, Schaller JE (2012) Dispatching heuristics for the single machine weighted quadratic tardiness scheduling problem. *Computers & Operations Research* 39(9):2223–2231
34. Vepsalainen AP, Morton TE (1987) Priority rules for job shops with weighted tardiness costs. *Management science* 33(8):1035–1047
35. Wang X, Tang L (2009) A population-based variable neighborhood search for the single machine total weighted tardiness problem. *Computers & Operations Research* 36(6):2105–2110

